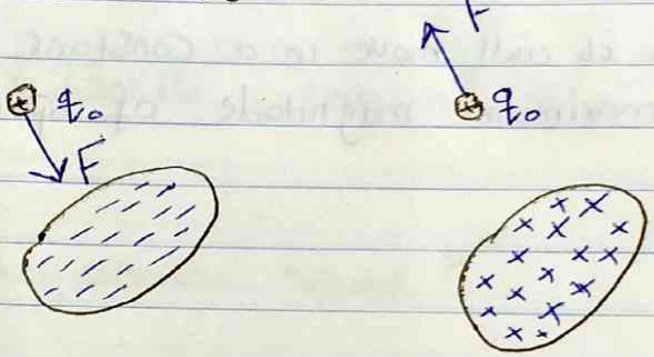
Chapter 22

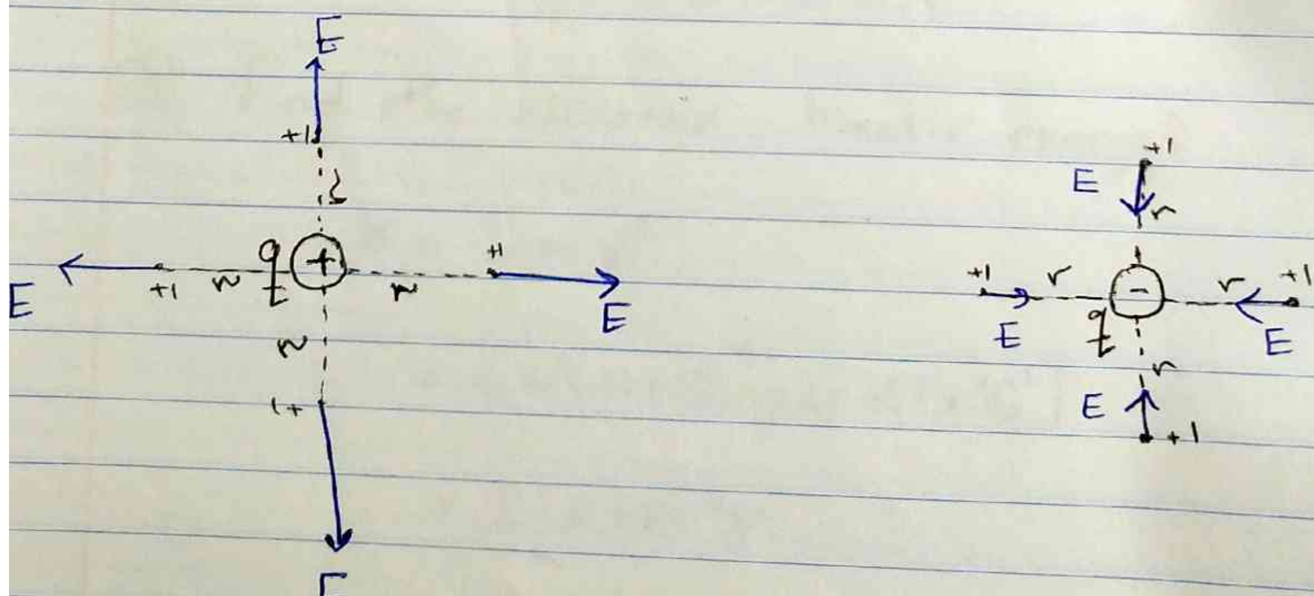
Electric fields

- **Electric field:** is the electric force acting on +1 C.

$$\vec{E} = \frac{\vec{F}}{q_0} \quad N/C$$

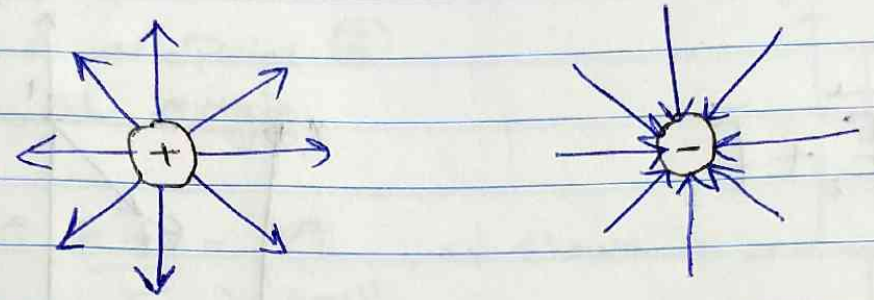


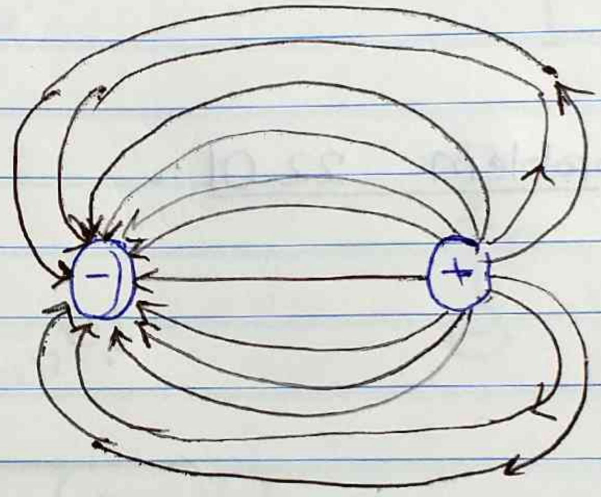
- **Electric field due to a point Charge:**

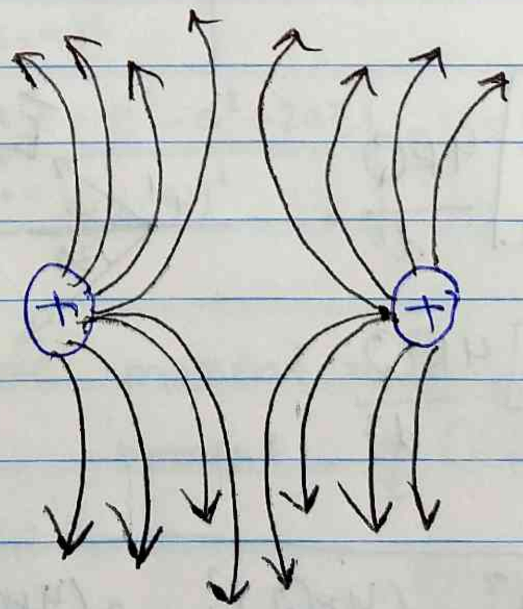


$$※\ E = \frac{kq}{r^2}\,\hat{r} \quad N/C$$

• Electric field line :-



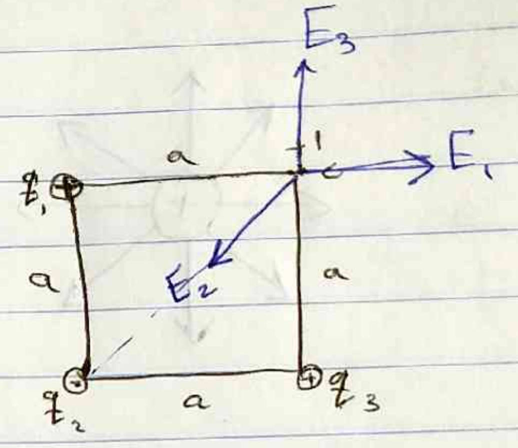




• $\vec{E}$ due to a set of Point Charges :-

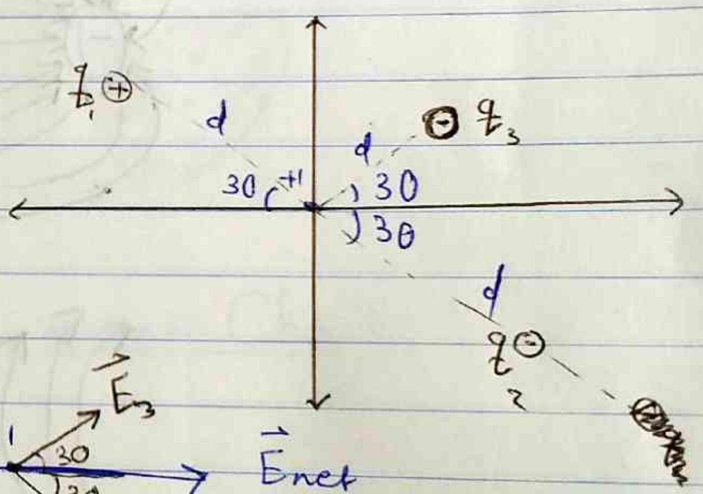
find $\vec{E}_{net}$ on C ?

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



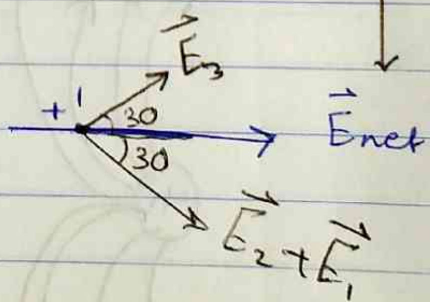
• Sample problem 22.01 :-

$q_1 = +2Q$
$q_2 = -2Q$
$q_3 = -4Q$



$$\vec{E}_1 = \frac{k\,2Q}{d^2}, \quad \vec{E}_2 = \frac{k\,2Q}{d^2} \quad \frac{4kQ}{d^2}$$

$$\vec{E}_3 = \frac{k\,4Q}{d^2} \quad \frac{4kQ}{d^2}$$

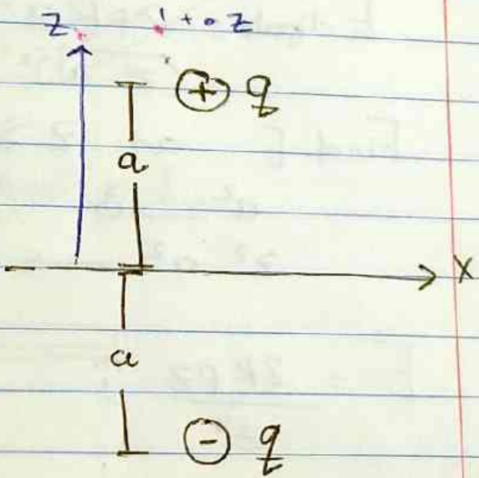


$$\vec{E}_{net} = \sqrt{\left(\frac{4kQ}{d^2}\right)^2 + \left(\frac{4kQ}{d^2}\right)^2 + 2\left(\frac{4kQ}{d^2}\right)^2 \cos 60}$$

$$\vec{E}_{net} = \sqrt{3}\,\frac{4kQ}{d^2} \text{ N/C}$$ in (+x) axis direction.

The electric field due to an electric Dipole:

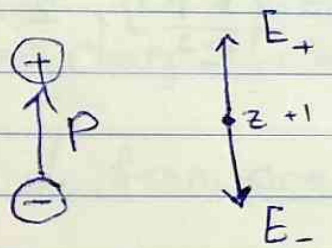
* Find E at a point (z) from the origin



$$E_+ = \frac{kq}{r^2} = \frac{kq}{(z-a)^2}, \text{ as shown}$$

$$E_- = \frac{kq}{r^2} = \frac{kq}{(z+a)^2}, \text{ as shown}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$



$$= \frac{kq}{(z-a)^2} - \frac{kq}{(z+a)^2}$$

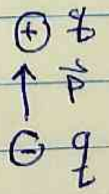
$$E = kq\left[\frac{(z+a)^2-(z-a)^2}{(z+a)^2(z-a)^2}\right]$$

$$= kq\left[\frac{z^2+a^2+2az-z^2-a^2+2az}{((z+a)(z-a))^2}\right] = kq\left[\frac{4az}{(z^2-a^2)^2}\right]$$

. let electric Dipole moment $= q\,d$ $\boxed{d=2a}$

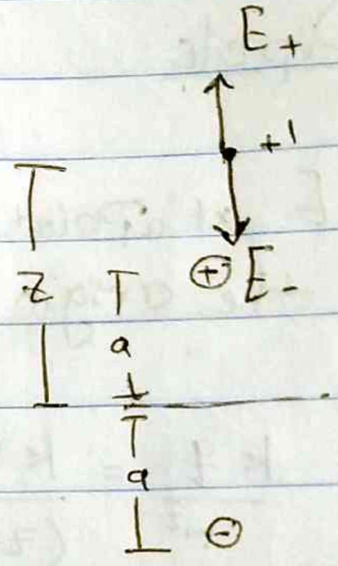
moment $= q\,(2a)$ C.m

. $\vec{p} = q\,(2a)$



. $E = \frac{2kpz}{(z^2-a^2)^2} = \frac{2kpz}{(z^2-a^2)^2}$

$\cdot\ \vec{E}$ due to an Electric Dipole:



$\cdot\ \vec{E}_{dipole} = \dfrac{2pKz}{(z^2-a^2)^2}\ \hat{j}$

Find E at $z \gg a$

$a^2 \longrightarrow 0$

$z^2 - a^2 \longrightarrow z^2$

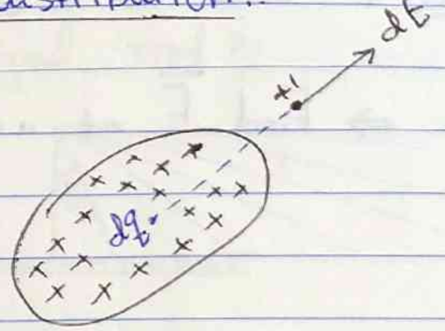
$$E = \frac{2Kpz}{z^4}\ \hat{j}$$

$$= \frac{2Kp}{z^3}\ \hat{j},\ z \gg a$$

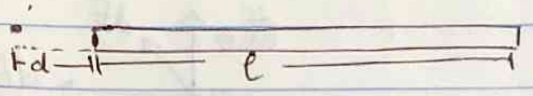
• $\vec{E}$ due to a continues Charge distribution:

$$dE = \frac{k\,dq}{r^2}$$

$$E = \int dE$$

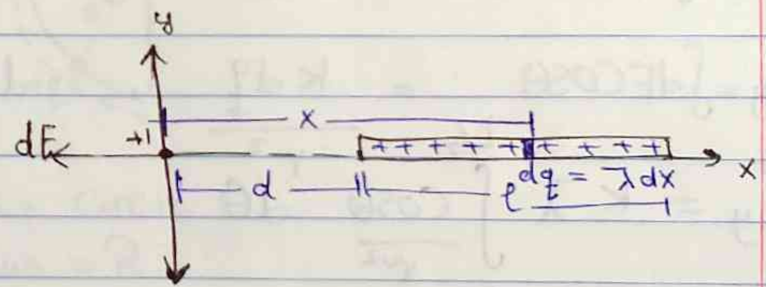


① $\vec{E}$ due to a Uniformaly Chargeel rod:



Charge rod → length = l
→ Charge = q
→ $\lambda = \frac{q}{l}$ C/m (linear charge dinsity)

⇒ find $\vec{E}$ at a point a distance d from one end?



$$dE = \frac{k\,dq}{x^2}$$

$$dE = \frac{k\,dx\,\lambda}{x^2}$$

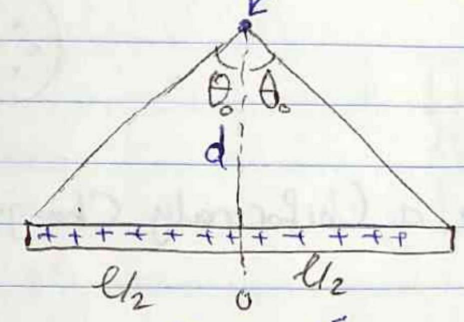
$$E = k\lambda \int_{d}^{l+d} \frac{dx}{x^2}$$

$$= k\lambda \left[-\frac{1}{x}\right]_{d}^{l+d}$$

$$= \frac{\lambda l}{\epsilon_0 4\pi d(l+d)}(-\hat{i}) = \frac{q}{4\pi\epsilon_0 d(l+d)}(-\hat{i})$$

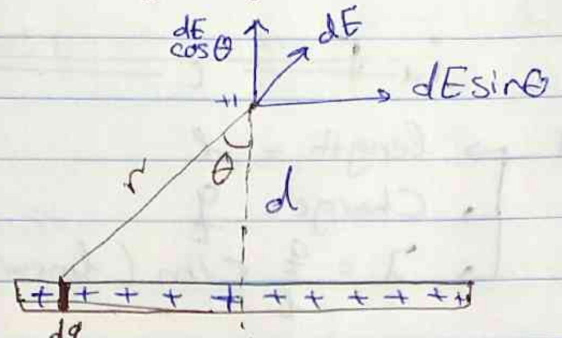
② $\vec{E}$ due to a Uniformly Charged rod:-

$\Rightarrow$ Find $\vec{E}$ at a distance (d) above the midpoint.



$$dE = \frac{k\,dq}{r^2}$$

$$dE_x = dE \sin\theta$$
$$dE_y = dE \cos\theta$$



$E_x = \int dE_x = 0$ from symmetry

$$E_y = \int dE\cos\theta \qquad = \frac{k\,dq}{r^2}, \qquad dq = \lambda\,dx$$

$$\Rightarrow E_y = k\lambda \int_{-L/2}^{L/2} \frac{\cos\theta}{r^2}\, d\theta$$

$$= k\lambda \int_{-\theta_0}^{\theta_0} \frac{\cos\theta\, \cancel{d}\sec^2\theta\, d\theta}{\sec^2\theta\; d^2}$$

$$= \frac{k\lambda}{d} \int_{-\theta_0}^{\theta_0} \cos\theta\, d\theta$$

$$= \frac{k\lambda}{d} \left[\sin\theta\right|_{-\theta_0}^{\theta_0} = \frac{2k\lambda}{d}\sin\theta_0$$

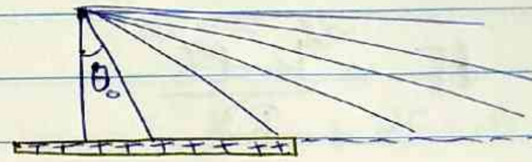
$$= \frac{\lambda\, L/2}{2\pi\epsilon_0 d\sqrt{d^2+\frac{L^2}{4}}}$$

$$\cos\theta = \frac{d}{r}$$
$$r = \frac{d}{\cos\theta}$$
$$r = \sec\theta\, d$$
$$r^2 = \sec^2\theta\, d^2$$
$$\tan\theta = \frac{x}{d}$$
$$x = d\tan\theta$$
$$dx = d\sec^2\theta\, d\theta$$

$$\sin\theta_0 = \frac{L/2}{\sqrt{d^2+\frac{L^2}{4}}}$$

• Find E for infine Charged rod?

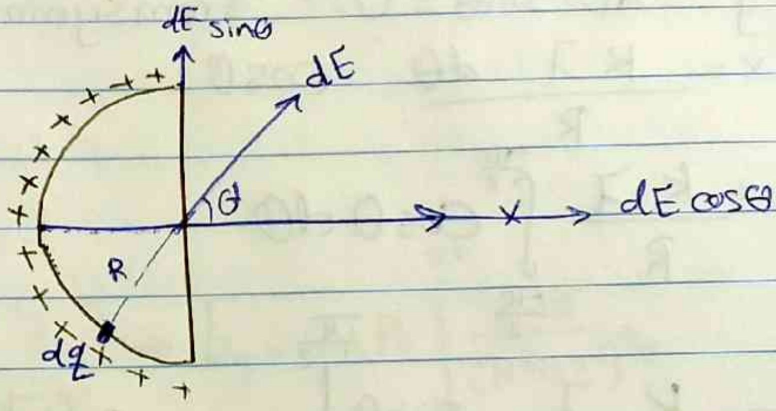
$\theta_0 \longrightarrow \frac{\pi}{2}$



$\sin\frac{\pi}{2} = 1$

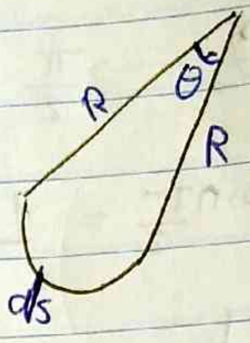
$$E = \frac{\lambda}{2\pi\epsilon_0 d}$$

③ $\vec{E}$ due to a Uniformly Charged arc:-



Arc →
- Semi - Circle
- reoduis $= R$
- length $= R\pi$
- Charge $= q$
- $\lambda = \frac{q}{R\pi}$ c/m

. Find $\vec{E}$ at the Center ?



$$dE = \frac{k\,dq}{R^2}$$

$$= \frac{k(\lambda ds)}{R^2}$$

$$dE = \frac{k\lambda R\,d\theta}{R^2} = \frac{k\lambda\,d\theta}{R}$$

$ds = R\,d\theta$

$dE_y = dE \sin\theta = 0$ from symm.

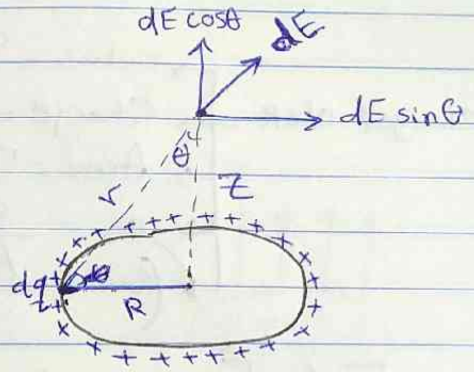
$$dE_x = \frac{k\lambda\,d\theta}{R}\cos\theta$$

$$E_x = \frac{k\lambda}{R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\,d\theta$$

$$E_x = \frac{k\lambda}{R}\sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2k\lambda}{R} = \frac{\lambda}{2\pi\epsilon_0 R}$$

④ $\vec{E}$ due to a Uniformly Charge Ring:

Ring
- $\rightarrow$ radius $= R$
- $\rightarrow$ Charge $= q$
- $\rightarrow$ length $= 2\pi R$
- $\rightarrow$ $\lambda = \frac{q}{2\pi R}$



• Find $\vec{E}$ a distance (Z) above the center?

$$dE = \frac{k\,dq}{r^2} = \frac{k\lambda\,ds}{R^2+Z^2} = \frac{k\lambda\,ds}{R^2+Z^2} = \frac{k\lambda R\,d\theta}{R^2+Z^2}$$

$$dE_x = dE\sin\theta$$

$$E_x = \int dE_x = 0$$

$$dE_y = dE\cos\theta$$

$$dE_y = k\frac{\lambda R\,d\theta}{R^2+Z^2}\;\frac{Z}{\sqrt{R^2+Z^2}} \Rightarrow E_y = k\lambda R\int_0^{2\pi}\frac{Z\,d\theta}{(R^2+Z^2)^{3/2}}$$

$$= \frac{k\lambda R Z}{(R^2+Z^2)^{3/2}}\int_0^{2\pi} d\theta \qquad \boxed{\lambda = \frac{q}{2\pi R}}$$

$$= \frac{2\pi k\lambda R Z}{(R^2+Z^2)^{3/2}} = \frac{k q Z}{(R^2+Z^2)^{3/2}}$$

• Find $\vec{E}$ on the center?

on the center $Z=0 \Rightarrow E=0$

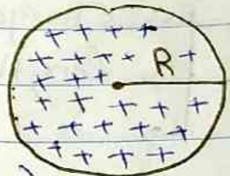
• Find $\vec{E}$ at $Z \gg R$? $\quad E = \frac{kq}{Z^2}$ « as a point » Charge

$$R^2 \longrightarrow 0$$

$$Z^2+R^2 \longrightarrow Z^2$$

⑤ $\vec{E}$ due to a Uniformly Charged disk :-

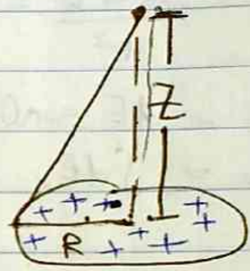
Charged disk → radius = R
→ Charge = q
→ Area = πR^2
→ $\sigma = \frac{q}{A}$ c/m^2
(Surface Charge density)



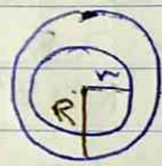
• Find $\vec{E}$ a distance Z above the Center?

dq = Charge on a ring
of radius = r
thickness = dr

$dq = \sigma \, dA = \sigma (2\pi r dr)$



$$(dE)_{ring} = \frac{K(\sigma 2\pi r dr) z}{(z^2 + r^2)^{3/2}}$$



$$E_{disk} = \int dE_{ring}$$

$$= K \sigma \pi z \int_0^R \frac{2r dr}{(z^2 + r^2)^{3/2}}$$

let $u = z^2 + r^2$
$du = 2r dr$

$$= K \sigma \pi z \int_0 \frac{du}{u^{3/2}}$$

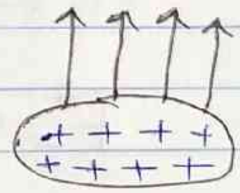
$$= -2K\sigma \pi z \, u^{-\frac{1}{2}} \Big| = -2K\sigma\pi z (z^2+r^2) \Big|_0^R$$

$$E_{disk} = 2K\sigma\pi \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right]$$

$$E_{disk} = 2k\sigma\pi\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]$$

$$E_{disk} = \frac{\sigma}{2\epsilon_0}\left[1-\frac{Z}{\sqrt{R^2+Z^2}}\right]$$

- Find dE when $R \gg Z$?



$$R \to \infty$$

$$\frac{Z}{\sqrt{R^2+Z^2}} \to 0$$

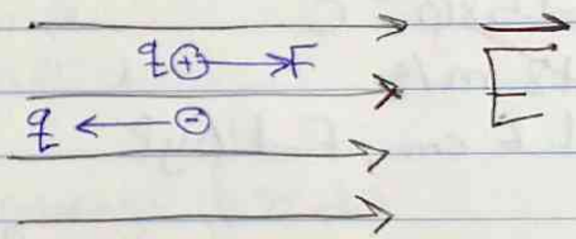
$\Rightarrow$ $E_{disk} = \dfrac{\sigma}{2\epsilon_0}$ for infinite Charged plan.

- <u>Motion of a free Charged Particle in a uniform electric filed:-</u>

$$\vec{F} = q\vec{E}$$

$$m\vec{a} = q\vec{E}$$

$$\vec{a} = \frac{q}{m}\vec{E} \text{ constant}$$



SO: we can use:

① $V_{2x} = V_{1x} + a_x t$

② $V_{2x}^2 = V_{1x}^2 + 2a_x \Delta x$

③ $\Delta x = V_{1x}t + \frac{1}{2}a_x t^2$

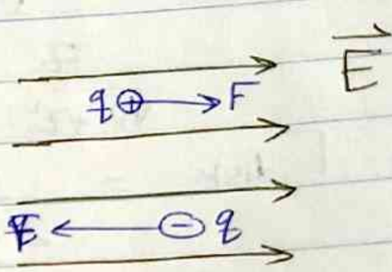
④ $\Delta x = \left(\dfrac{V_{1x} + V_{2x}}{2}\right) t$

• Motion of a free Point charge in a uniform $\vec{E}$:-

→ uniform $\vec{E}$ constant in ↗ magnitude ↘ direction

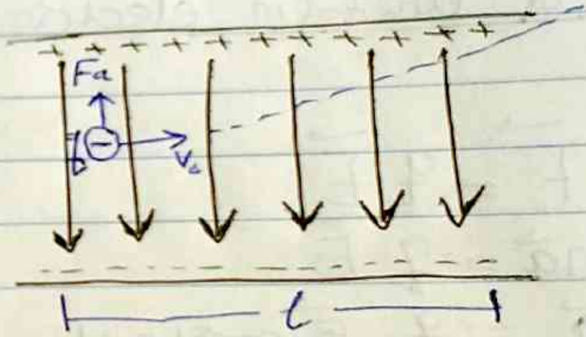
$$\vec{F} = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q\vec{E}}{m}$$



. Sample problem 22.04 :

. $\vec{E} = 1.4 \times 10^{6}$ N/C
. $m = 1.3 \times 10^{-10}$ kg
. $q = -1.5 \times 10^{-13}$ C
. $V_0 = 18$ m/s
. $l = 1.6$ cm , Find Δy?



$$\rightarrow \vec{F} = q\vec{E} = m\vec{a} \rightarrow a_y = \frac{qE}{m} = 1.6 \times 10^{3}\ m/s^2$$

$$\rightarrow \Delta y = \cancel{V_{oy}}\, t + \frac{1}{2} a_y t^2 \rightarrow \Delta y = \frac{1}{2} a_y t^2$$

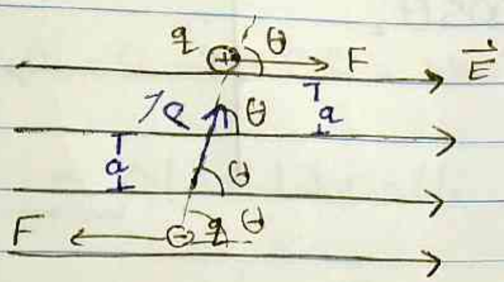
$$V = \frac{d}{t} \Rightarrow 18 = \frac{1.6 \times 10^{-2}}{t}$$

$$\Rightarrow t = 0.088 \times 10^{-2}\ s$$

$$\Rightarrow \Delta y = \frac{1}{2} a_y t^2 = \cancel{6.3 \times}\ 10^{-1} = 0.00704\ m$$

0.0704

· An electric Dipole a Uniform $\vec{E}$:



$d = 2a$

$p = qd = q\,2a$

$\vec{p}\ (-) \rightarrow (+)$

· $F_+ = F_- = qE$

$\vec{P}$ will rotate $\Rightarrow$ $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau_+ = (-)\,aqE\sin\theta$, clockwise

$\tau_- = (-)\,aqE\sin\theta$, Clock wise

$\vec{\tau}_{net} = -2aqE\sin\theta$, clock wise

$= -pE\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}\ \sin\theta$ N.m.

· W done by $\vec{E}$ in rotating $= \int_{\theta_i}^{\theta_f} \tau\, d\theta$

$= \int_{\theta_i}^{\theta_f} -pE\sin\theta\, d\theta$

$= -pE\left[-\cos\theta\right]_{\theta_i}^{\theta_f}$

· $W_E = pE\cos\theta_f - pE\cos\theta_i$

$q\vec{E}$ is a conservative force:-

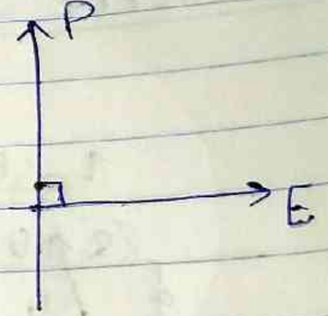
$W_E = -\Delta U$, $\Delta U = U_f - U_i = -W$

$U_f - U_i = pE\cos\theta_i - pE\cos\theta_f$

$U = 0$ at $\theta = 90°$

$$U_f - 0 = 0 - pE\cos\theta_2$$

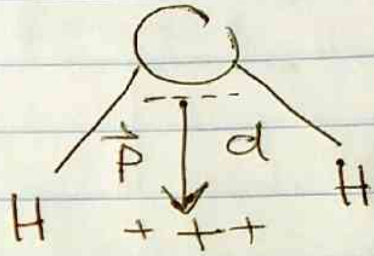
$$U_f = -pE\cos\theta_f$$



$$W_{F_{external}} = \Delta U$$

$$U = -\vec{P}\cdot\vec{E} \text{ Joul.}$$

- Sample problem:



$P = 6.2 \times 10^{-30}$ cm

$q = 10e$

(a). Find d ?!

$$d = \frac{P}{q} = \frac{6.2\times10^{-30}}{10(1.6\times10^{-19})}$$

$$d = 3.9 \times 10^{-12} \text{ m} \qquad = 3.9 \text{ Pm}$$

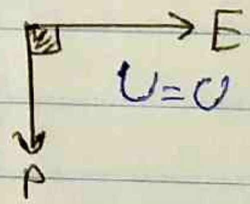
(b) . If $\vec{E} = 1.5\times10^{4}$ N/C acts On H_2O, Find τ max ?

$$\vec{\tau} = \vec{P}\times\vec{E}$$

$$\vec{\tau} = pE\sin\theta$$

$$\vec{\tau}_{max} = pE \ , \ \theta = 90^\circ$$

$$= 9.3\times10^{-26} \text{ m.N}$$



. minumum when $\theta = 0 \Rightarrow U = 1$

(c) W_{done} by external agent?

$$\theta_i = 0 \longrightarrow \theta_f = 180^\circ$$

① $\longrightarrow \vec{E}$
$\longrightarrow \vec{P}$

$$W_{F_{external}} = \Delta U = U_f - U_i$$

② $\vec{p} \longleftarrow \quad \longrightarrow \vec{E}$

$$W_{F_{external}} = (-PE\cos 180) - (-PE\cos 0)$$
$$= PE + PE$$
$$= 2pE$$
$$= 1.9 \times 10^{-25}\ J$$